

11.1 Mathematical Patterns
Algebra 2B

Name _____
Date _____ Per _____

Arithmetic: the **difference** between consecutive terms is constant (common difference d).

Geometric: the **ratio** between consecutive terms is constant (common ratio r).

Recursive Formula: defines the terms in a sequence by relating each term to the term before it.

For example: **Arithmetic:** $a_n = a_{n-1} + d$

Geometric: $a_n = a_{n-1} \cdot r$

→ To find a specific term given the recursive formula, you must be given the previous term.

Explicit Formula: defines the n th term in terms of n .

For example: **Arithmetic:** $a_n = a_1 + (n-1)d$

Geometric: $a_n = a_1 \cdot r^{n-1}$

Ex. #1 Find the first 5 terms of the sequence (recursive):

a) $a_n = a_{n-1} - 6; a_1 = 12$

12, 6, 0, -6, -12

b) $a_n = \frac{1}{3}a_{n-1}; a_1 = 12$

12, 4, $\frac{4}{3}$, $\frac{4}{9}$, $\frac{4}{27}$

Ex. #2 Find the first 5 terms of the sequence (explicit):

a) $a_n = 3n - 1$

2, 5, 8, 11, 14

b) $a_n = n(n-1)$

0, 2, 6, 12, 20

Ex. #3 Describe the following sequences as arithmetic, geometric, or neither. Then, write the recursive and explicit formula for each:

a) 4, 8, 12, 16, ...

Arithmetic

Recursive

$$\begin{aligned} a_n &= a_{n-1} + 4 \\ a_1 &= 4 \end{aligned}$$

Explicit

$$\begin{aligned} a_n &= 4(n-1) + 4 \\ a_n &= 4n \end{aligned}$$

b) 16, 8, 4, 2, ...

Geometric

R

$$\begin{aligned} a_1 &= 16 \\ a_n &= \frac{a_{n-1}}{2} \end{aligned}$$

E

$$a_n = 16\left(\frac{1}{2}\right)^{n-1}$$

c) 4, 9, 16, 25, ...

Neither

R

$$\begin{aligned} a_1 &= 4 \\ a_n &= (\sqrt{a_{n-1}} + 1)^2 \end{aligned}$$

E

$$a_n = (n+1)^2$$

d) -1, 1, -1, 1, ...

Geometric

R

$$\begin{aligned} a_1 &= -1 \\ a_n &= a_{n-1}(-1) \end{aligned}$$

E

$$a_n = (-1)^n$$

Classify each sequence as arithmetic, geometric, or neither. Then, find the next two terms:

1. 80, 77, 74, 71, 68, ... *A*

65, 62

2. 4, 8, 16, 32, 64, ... *G*

128, 256

3. 1, 2, 6, 24, 120, ... *N*

720, 5040

4. 100, 10, 1, 0.1, 0.01, ... *G*

.001, .0001

5. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \dots$ *G*

$\frac{1}{64}, \frac{1}{128}$

6. 0, 1, 0, $\frac{1}{3}$, 0, $\frac{1}{5}$, ... *N*

0, $\frac{1}{7}$

Write a recursive formula for each sequence. Then, find the next term:

7. 43, 41, 39, 37, 35, ...

$a_1 = 43$
 $a_n = a_{n-1} - 2$ (33)

8. 6, 1, -4, -9, ...

$a_1 = 6$
 $a_n = a_{n-1} - 5$ (-14)

9. 144, 36, 9, $\frac{9}{4}$, ...

$a_1 = 144$
 $a_n = \frac{1}{4} a_{n-1}$ ($\frac{9}{16}$)

Write an explicit formula for each sequence. Then, find the next term:

10. 4, 5, 6, 7, 8, ...

$a_n = 4 + (n-1)$
 $a_n = 3 + n$ (9)

11. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \dots$

$a_n = \left(\frac{1}{2}\right)^n$ ($\frac{1}{64}$)

12. 2, 5, 10, 17, 26, ...

$a_n = n^2 + 1$ (37)

Decide whether each formula is *explicit* or *recursive*. Then, find the first four terms:

13. $a_n = 2a_{n-1} + 3; a_1 = 3$

R

3, 9, 21, 45

14. $a_n = (n-5)(n+5)$

E

-24, -21, -16, -9

15. $a_n = \frac{1}{2}n(n-1)$

E

0, 1, 3, 6

16. $a_n = a_{n-1} - 17; a_1 = 340$

R

340, 323, 306, 289

17. $a_n = 2n^2 + 1$

E

3, 9, 19, 33

18. $a_n = \frac{1}{2}a_{n-1}; a_1 = -4$

R

-4, -2, -1, $-\frac{1}{2}$

Find the next two terms in the sequence. Then, write a recursive or explicit formula for the n th term. Identify each formula as *recursive* or *explicit*.

19. 5, 8, 11, 14, 17, ...

20, 23
R $a_n = a_{n-1} + 3$
 $a_1 = 5$

20. 1, 8, 27, 64, 125, ...

216, 343
E $a_n = n^3$

21. 49, 64, 81, 100, 121, ...

144, 169
E $a_n = (n+6)^2$